CPS296.2 Advance Topics in CPS: Mesh Generation

Homework # 4

Due date: October 30, Wednesday, the beginning of the class.

Credits: 10 full + 5 bonus

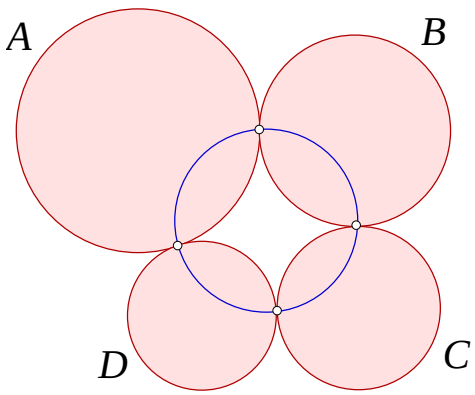
1. **(five credits)** A set B of unit disks in $\mathbb{R}^2$ is a packing if their interiors are pairwise disjoint. Let S be the set of centers. The density of B is

$$\rho(B) = \frac{area(convHull(S) \cap \bigcup B)}{area(convHull(S))}$$

   (a) Prove that each unit disk touches at most 6 other unit disks in a packing.

   (b) What is the density of the hexagonal grid, where each disk touches 6 others?

   (c) Argue that no packing has higher density than that of the hexagonal grid.

2. **(five credits)** Prove that in a packing of unit balls in $\mathbb{R}^3$ any one ball can touch at most 14 others. [Actual upper bound is 12, which you can work on proving for extra credits.]

3. **(five credits)** Let A, B, C, and D be four non-overlapping disks such that A and B, B and C, C and D, and D and A are tangent to each other. Prove that these four tangency points are co-circular.