CPS296.2 Advance Topics in CPS: Mesh Generation

Homework # 3

Due date: October 9, Wednesday, the beginning of the class.

Credits: 10 full + 1 bonus

1. **(four credits)** The *aspect ratio* of a simplex is the ratio of its circumradius to inradius. Show that this quality measure is equivalent to smallest angle measure, i.e.,
   (a) a lower bound on the smallest angle implies an upper bound on the aspect ratio;
   (b) an upper bound on the aspect ratio implies a lower bound on the smallest angle.

2. **(three credits)** The body centered cube (BCC) lattice is the set of points (i, j, k), $(i + 1/2, j + 1/2, k + 1/2)$ for $i, j, k \in \mathbb{Z}$. Delaunay triangulation of BCC lattice consists of congruent copies of a single tetrahedron. Determine all the metric properties of this tetrahedron: volume, areas of triangles, length of edges,face angles, dihedral angles, and solid angles.

3. **(four credits)** Let $\mathbb{I}^3 = [0, 1] \times [0, 1] \times [0, 1]$ be the unit cube in $\mathbb{R}^3$ and consider a triangulation K of $\mathbb{I}^3$ whose only vertices are the 8 corner points of the cube.
   (a) Show that every such K has at most 6 tetrahedra.
   (b) Show that every such K has at least 5 tetrahedra.
   (c) Two triangulations K_1 and K_2 are *isomorphic* if $\exists$ a bijection $\beta : Vertices(K_1) \to Vertices(K_2)$ such that $ConvHull(T) \in K_1$ iff $ConvHull(\beta(T)) \in K_2$. Enumerate all pairwise non-isomorphic triangulations of unit cube (with no Steiner points).